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Abstract

To determine to what extent children of preschool age comprehend the meaning of logical connectives, 64 5- and 6-year-olds were told to hand differently colored and shaped wooden blocks to an experimenter. The commands involved various English idioms used for conjunction (e.g. both black and round), disjunction (either black or round), and negation (not round). Analysis of the results indicated that socioeconomic status was the most telling factor, with disadvantaged children performing poorest. In addition, the older children performed better than the younger ones, but sex differences were not significant. The data further showed that disjunction was by far the most difficult operation for children and that the idiom in which the concept was expressed was a significant factor. A predictive regression model was developed from the data. A second experiment was performed with 112 4- to 6-year-olds. This experiment was modeled after the first except that type of connective, type of idiom, and order of commands were varied. The type of connective had the greatest effect but negation was also an important variable. (MH)

# YOUNG CHILDREN'S COMPREHENSION OF LOGICAL CONNECTIVES

by

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Young Children's Comprehension of Logical Connectives[1]

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The development of children's understanding of logical connectives has been discussed by a number of people over the past decade and a half. Without attempting to review this literature in detail, we cite Furth and Youniss (1965), Hill (1961), Inhelder and Matalon (1960), Inhelder and Piaget (1964), McLaughlin (1963), Piaget (1957), Suppes (1965), and Youniss and Furth (1964, 1967). Nevertheless, none of these studies reports the extent to which children of preschool age show comprehension of the meaning of the logical connectives in a well-defined experimental situation.

The importance of understanding the extent and limitations of children's mastery of the logical connectives is evident for any cognitive theory of development. The recent work in psycholinguistics, emphasizing the complex nature of the grammar and semantics of the language of children, has provided further impetus for seeking such understanding.

It seems clear that the development of a better theory about children's behavior and the changes in that behavior with age requires much more detailed information about their linguistic habits and competence than we now have. The present study, which consists of two closely related experiments, is meant to contribute to the accumulation of such systematic information.

The data of the experiments have been analyzed in terms of several specific regression models to provide a deeper insight into what aspects

of comprehension of sentential connectives are most difficult. In addition, a more detailed mathematical model is applied to the data of Experiment 2 to predict the distribution of actual responses, not just the proportion of response errors.

The formal relations between various English idioms expressing conjunction, disjunction, and negation, and the set-theoretical operations of intersection, union, and complementation are not deeply explored in this paper, but our assumptions about these connections are obvious and uncontroversial. Deeper investigation of these linguistic and semantical matters seems desirable as part of any further extensive study of children's comprehension of logical connectives.

## EXPERIMENT 1

The primary aim of the first experiment was to investigate the extent to which children between the ages of four and six comprehend the logical connectives of conjunction, disjunction, and negation. It was also anticipated that the exact formulation of the idiom in terms of which the connectives were expressed would affect the results. Consequently, a second aim was to investigate the relative ease or difficulty of various idioms used to express the connectives. A third, subsidiary aim of the experiment was to examine the effects of sex, age, and socioeconomic status on the performances of the children in comprehending the meaning of the connectives.

### Method and Procedure

#### Experimental Design

The differences in performance as a function of the type of logical connective and idiom were examined in a within-subject design. A

2 x 2 x 2 factorial design was used to examine the between-subject effects of age, sex, and socioeconomic status (SES).

## Subjects

Sixty-four subjects participated in the experiment. Thirty-two kindergarten children between the ages of 5.7 and 6.7 years were drawn from two sources: a middle-class elementary school and a school in a disadvantaged area. The other 32 children were between the ages of 4.5 and 5.4 years and attended either a preschool headstart class or a middle-class nursery school. Eight boys and 8 girls were tested from each of the four sources. The children from the preschool headstart class and from the school in a disadvantaged area were considered culturally deprived by the standards of the Office of Economic Opportunity.

## Experimental Materials

Eighteen wooden blocks were used. Each block had two salient properties: shape (star, circle, or square) and color (red, green, or black). Each combination of color and shape was represented by two blocks. The blocks were approximately 3-1/2 inches square and 1/2 inch deep.

## Procedure

The children were pretested to ensure that they could identify the elementary properties (color and shape) of the blocks. Three children were eliminated at this stage, but were replaced by others so that 64 subjects were tested.

Each subject was tested individually. After some preliminary commands, the subjects received 12 test commands to hand various blocks to the experimenter. The subjects were told to give all the blocks

asked for, and none of the others. Throughout the session the experimenter behaved in a positively reinforcing manner.

The commands were stated with as much inflection as possible. For example, command 11 was expressed as "the things that are green-or-square," with the hyphenated words spoken as a coherent unit. Words were stressed and pauses were used to heighten the effect of the logical connectives. Each command was repeated several times.

The commands were as follows:

1. Give me the green stars.
2. Give me the red things and the square things.
3. Give me the things that are black, but not round.
4. Give me all the red things, and then everything else, but not the stars.
5. Give me all the things that are black and square.
6. Give me the green things, or, the round things.
7. Give me the stars that are red.
8. Give me the things that are black and not square.
9. Give me all the things that are green, and then everything else but not the stars.
10. Give me the black things that are round.
11. Give me the things that are green or square.
12. Give me the things that are not round but are red.

Seven commands--commands 1, 3, 5, 7, 8, 10, and 12--tested the conjunction or the intersection of two sets. Three of these, namely, commands 3, 8, and 12, also used negation; i.e., they asked for the intersection of sets when one of the sets was a complement. To investigate disjunction or the union of sets, four alternate forms of the

commands were given. These were commands 2, 4, 9, and 11; of these, commands 4 and 9 involved negation or complementation. One command, namely, command 6, used the "exclusive-or" connective. The commands not using negation will be called positive commands; the commands using "not" will be called negative commands.

## Results and Discussion

Three types of analyses were performed: one on the differences in performance between the various groups of subjects, one on the types of responses made to the different connectives and the different idioms used to express the connectives, and one on the predictive worth of a regression model.

### Group Differences

To evaluate the contribution of age, sex, and socioeconomic status (SES), five three-way analyses of variance on the number of correct responses were carried out: one each on the total score, the score to conjunction commands, to disjunction commands, to negation commands, and to the exclusive-or command. In Table 1, the significant results

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Insert Table 1 about here

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from these analyses are presented. In four of the five analyses, SES was a significant variable with children from culturally deprived homes consistently making fewer correct responses than children from advantaged homes. Age also was important in comprehending the connectives of conjunction and negation, with the older children making more correct responses than the younger children. However, it should be noted that

TABLE 1

Summary of Significant Variables from Five Three-Way Analyses of Variance on Correct Responses in Experiment 1

| Analysis | Significant variables | MS | df | F |
|---|---|---|---|---|
| 1. Total commands | SES | 76.6 | 1 | 33.0** |
| | (Error) | 2.3 | 56 | |
| 2. Conjunction commands | SES | 34.5 | 1 | 28.3** |
| | Age | 15.1 | 1 | 12.3** |
| | (Error) | 1.2 | 56 | |
| 3. Disjunction commands | SES | 5.1 | 1 | 10.5** |
| | Age × Sex × SES | 2.3 | 1 | 4.7* |
| | (Error) | 0.5 | 56 | |
| 4. Negation commands | SES | 40.6 | 1 | 35.6** |
| | Age | 8.3 | 1 | 7.2** |
| | (Error) | 1.1 | 56 | |
| 5. Exclusive-Or commands | — | | | |

* $p < .05$

** $p < .01$

the connectives of conjunction and negation were not independent. Sex did not affect performance differentially. Finally, only 1 of a possible 20 interactions was significant. It tentatively may be concluded that the three main effects are independent of one another. The fact that socioeconomic status was a uniformly more significant variable than age is to be emphasized.

Response Distributions

The number of correct responses to a logical connective is an estimate of the difficulty of the operation. The rank-order of the difficulty of the binary connectives from least to most difficult is as follows: conjunction (71 per cent correct), exclusive-or (67 per cent), and disjunction (11 per cent). Significant differences are found between conjunction and disjunction ($z = 9.1$, $p < .001$), between exclusive-or and disjunction ($z = 8.2$, $p < .001$), but not between conjunction and exclusive-or. For the combined connectives negation substantially increases the difficulty of the commands ($z = 3.8$, $p < .01$). However, it does not affect the rank-order of the different connectives, and from least to most difficult the order is as follows: positive conjunction (81 per cent correct), exclusive-or (67 per cent), conjunction-negation (56 per cent), positive disjunction (18 per cent), and disjunction-negation (6 per cent).

The errors that the children made to the commands indicate the source of difficulty in understanding connectives. In Table 2 the notation

Insert Table 2 about here

TABLE 2

Legend of Notation Used to Describe Subjects' Responses in Both Experiments

| Symbol | Definition | Example: X = red, Y = square |
|---|---|---|
| X | The set of elements with attribute X. | All the red blocks. |
| $X \cap Y$ | Conjunction. The intersection of sets X and Y. Each object has both attributes X and Y. | The red squares. |
| $X \cup Y$ | Disjunction. The union of sets X and Y. Each object has at least one of the attributes X and Y. | The red blocks and the square blocks. |
| X or Y | The exclusive-or (All members of the set X or all members of the set Y, but not both). | The set of red blocks or the set of square blocks. |
| $\overline{X}$ | Negation. The complementary set of X. The not-X objects. | Green blocks and black blocks. |
| $\frac{1}{n} X$ | The incomplete set of X objects. | Five or less (of the six) red blocks. |
| $\frac{1}{n} (X \cup Y)$ | The incomplete set of blocks belonging to the union of X and Y, where members of both X and Y are represented. | Five or less of the red <u>and</u> five or less of the square blocks. |
| $\frac{1}{n} \overline{X}$ | The incomplete complementary set where members of both subsets in the complementary set are represented. | Some, but not all of the green blocks and black blocks. |
| Misc. | Miscellaneous--any response not defined by the above categories. | |

used to describe the responses is presented. In Table 3 the response distribution for the 12 commands is shown. For the positive conjunction

Insert Table 3 about here

commands, three of the four commands (1, 7, and 10) have similar distributions. The responses to the three conjunction-negation commands (3, 8, and 12) also show a similar distribution to one another. The distribution of the responses to the two positive disjunction commands (2 and 11) are not alike, a difference probably due to the different idioms used.

In general, an inverse relationship exists between number of blocks for a correct response and performance.

When the command was for the set $X \cap \overline{Y}$, giving the intersection of one set with only a part of the complementary set was the most frequent error. It is not clear whether the difficulty was in identifying the extension of the complementary set or in the operation of intersection. Some recent evidence (Feldman, 1968) indicates that 4- to 6-year olds have difficulty in being exhaustive with the complementary set, which seems to suggest that the complementation caused the difficulty in these commands rather than the intersection. Another frequent error was to give the first-mentioned set, as may be seen in Table 3.

The negative disjunction commands (4 and 9) were difficult for the subjects, and many errors were made. The first-mentioned set appears in 9 per cent of the responses. Observations of the children revealed that many of them had genuine conflict over where to place the blocks that belonged to $X \cap Y$; for example, in command 4, placement of the red stars when the red things and the not-stars were requested. Many children

TABLE 3

Distribution of Responses to the Twelve Commands of Experiment 1.

Data Expressed as Percentages.

| Connective | Command | Response | | | | |
|---|---|---|---|---|---|---|
| | | $X \cap Y$ | $X \cup Y$ | X | Y | Misc. |
| $X \cap Y$ | 1 | 97* | 0 | 0 | 0 | 3 |
| | 5 | 42* | 13 | 22 | 11 | 12 |
| | 7 | 96* | 0 | 2 | 2 | 0 |
| | 10 | 92* | 0 | 3 | 2 | 3 |
| $X \cup Y$ | 2 | 11 | 33* | 33 | 0 | 23 |
| | 11 | 47 | 3* | 25 | 6 | 19 |
| X or Y | 6 | 9 | 6 | 45* | 23* | 17 |

| Connective | Command | Response | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $X \cap \overline{Y}$ | $X \cup \overline{Y}$ | $X \cap \frac{1}{n} \overline{Y}$ | $X \cap Y$ | X | $\overline{Y}$ | Misc. |
| $X \cap \overline{Y}$ | 3 | 56* | 0 | 23 | 2 | 14 | 0 | 5 |
| | 8 | 56* | 0 | 25 | 3 | 8 | 0 | 8 |
| | 12 | 56* | 0 | 30 | 3 | 5 | 0 | 6 |
| $X \cup \overline{Y}$ | 4 | 30 | 5* | 2 | 0 | 12 | 43 | 8 |
| | 9 | 42 | 3* | 5 | 0 | 6 | 38 | 6 |

*correct response

first included the red stars (in response to command 4) with the objects they gave to the experimenter, but then verbalized "But these are stars, and you said the red things and the things that are not stars," and with this comment removed the red stars. Although an order of selection clearly was suggested by the phrase "and then everything else..." very few children selected their blocks in this manner. Most children picked up each block as it came to hand and apparently tested it against a memorized version of the command.

For the exclusive-or command the correct response category is probably inflated, for the first-mentioned set is a highly probable component of the response irrespective of the connective used.

Idiom. Table 3 shows that the form of the idiom used to express a particular logical connective affected the difficulty of the command for the subjects. In the case of conjunction, the idiom of command 5, "Give me all the things that are X and Y," was especially difficult. The other three (commands 1, 7, and 10) were quite easy, as reflected in the high proportion of correct responses. As for disjunction, only one idiom was understood with any success and that was "Give me the X things and the Y things" (command 2). The idiom of command 11 was obviously very difficult for the children.

## Regression Models

The discussion of regression models follows Suppes, Hyman, and Jerman (1967). The main task is to identify the factors that contribute to the difficulty of the commands. Factors to be examined include variation in the connective, the idiom, and the order of the properties. As a matter of notation, the jth factor of command i in a given set of commands is

denoted by $f_{ij}$. The statistical parameters to be estimated from the data are the weights $\alpha_j$ attached to each factor. We emphasize that the factors identified and used here are not abstract constructions from the data. Rather, they are always objective factors identifiable by the experimenter in the commands themselves, independent of any data analysis. Which factors turn out to be important is a matter of the estimated weights $\alpha_j$. Let $p_i$ be the observed proportion of correct responses to command i for the group of subjects. The central task of the model is to predict these observed proportions. The natural linear regression model in terms of the factors $f_{ij}$ and the weights $\alpha_j$ is simply

$$p_i = \sum_j \alpha_j f_{ij} + \alpha_0 .$$

All the factors $f_{ij}$ used are 0,1-variables that take the value 1 when present and 0 when absent. The four connective factors were $C_1$ = disjunction, $C_2$ = conjunction, $C_3$ = negation, and $C_4$ = exclusive-or. The second type of factor considered was the form of the idiom used. Four idiom factors were used, namely, $I_1$ = Give me the things that are X..., $I_2$ = Give me all the X things and everything else..., $I_3$ = Give me the X things and/or the Y things..., and $I_4$ = Give me the X (things) that are Y. This classification included 11 of the 12 commands. The command not included was the first one, for it did not fit any of the four categories $I_i$. The final factor used was an order variable, $D_i$, which took the value 1 when shape was the first-mentioned dimension.

A standard stepwise, multiple linear regression program was used to obtain regression coefficients, multiple correlation R, and $R^2$.

For the regression equation

$$p_i = .89 - .31C_1 + .08C_2 + .13C_3$$

$$- .55I_1 - .65I_2 - .25I_3 - .05I_4 + .03D_1$$

the multiple R is .995, with a standard error of .0203. Figure 1 shows the predicted and observed success ratios. Although the fit is good, it

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Insert Figure 1 about here
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must be remembered that 9 parameters are being estimated (and thus 8 structural variables are being used) to make 11 predictions.

If we reduce the number of variables in the regression equation, the problem of interpreting the coefficients is made easier and the reduction in multiple R and $R^2$ is not very great. Considering only the first four variables that entered in the stepwise regression, the equation becomes

$$p_i = .64 - .26C_1 + .29C_2 - .39I_1 - .32I_2$$

with a multiple R of .991, and a standard error of estimate of .0579.

Several features of the regression coefficients should be noted. Disjunction commands are difficult, and conjunction commands are easy. Negation does not enter into this regression equation, and the predictions are satisfactory without this variable. Figure 2 shows the predicted and observed success ratios.

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Insert Figure 2 about here
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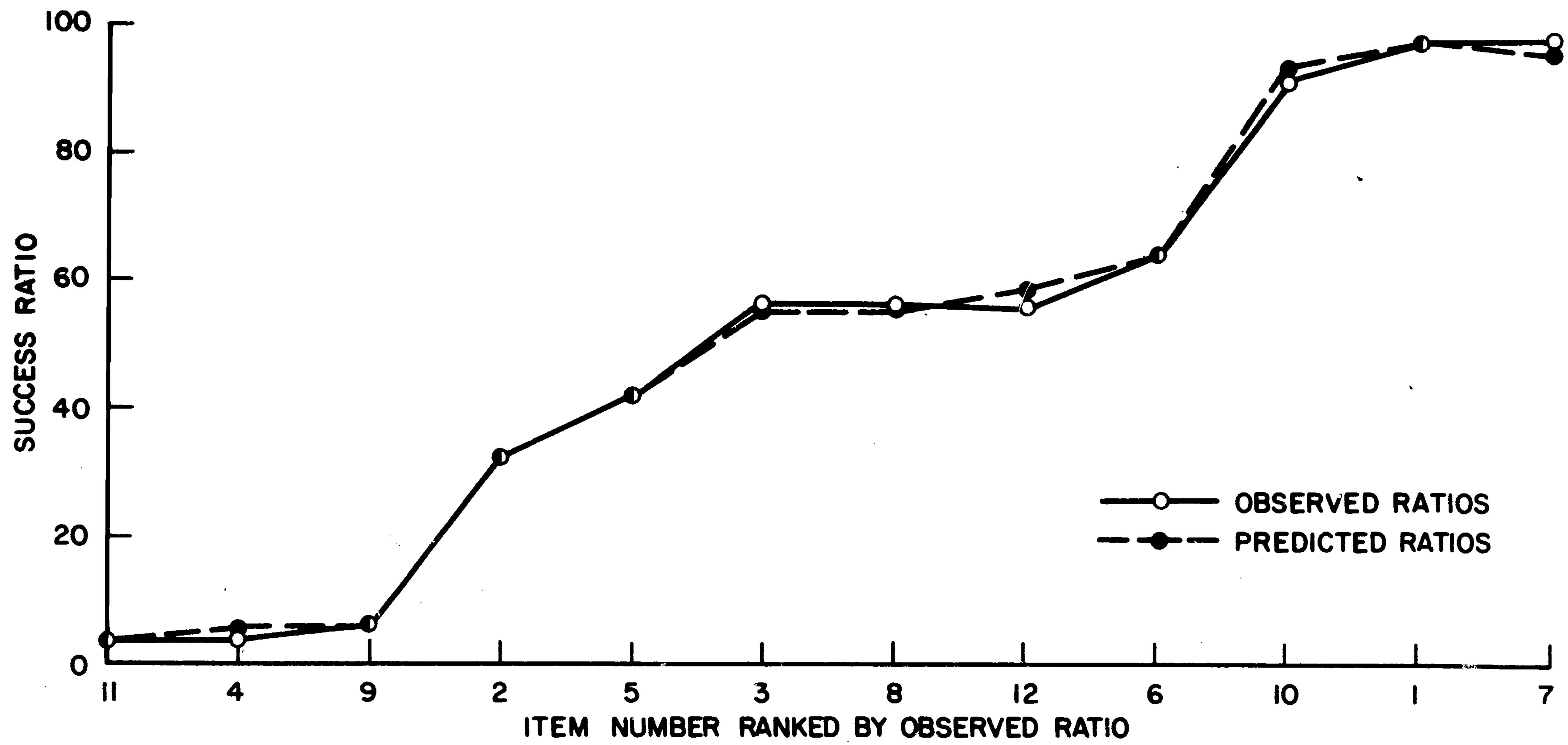


Figure 1. Predicted and observed success ratios with eight variables in the regressicn equation, Experiment 1.

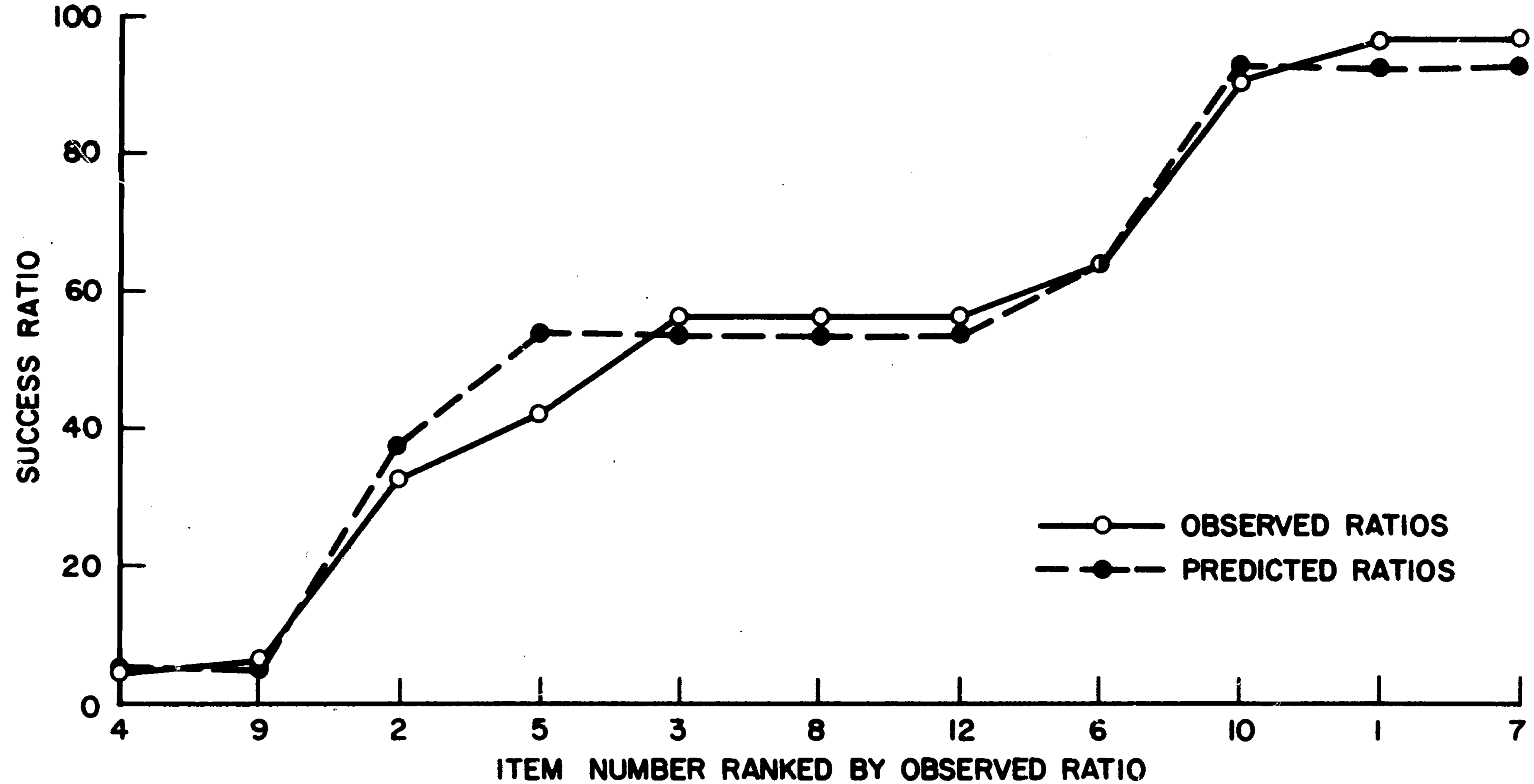


Figure 2. Predicted and observed success ratios with four variables in the regression equation, Experiment 1.

## EXPERIMENT 2

Both regression analyses in Experiment 1 show the significance of connective and idiom variables. In order to investigate further the role of idioms in children's understanding of sentential connectives, we performed a second experiment with a new group of subjects in which the connectives and idioms were standardized in a manner described below.

### Method and Procedure

#### Experimental Design

The subjects were divided into four groups, with age and sex equated across the groups. Each group was given the same set of 12 commands, with the order of the commands different for each group. Thus, type of connective and type of idiom were within-subject variables and order of commands was a between-subject variable. Each subject was tested individually.

#### Subjects

The 112 subjects between 4.6 and 6.0 years of age were drawn from the Stanford Nursery School and from the kindergarten classes of local elementary schools.

#### Experimental Materials

These were the same as in Experiment 1.

#### Procedure

The task and the instructions were similar to those described in Experiment 1. However, the idiom and the connectives were standardized. Three forms of idioms were used. They were:

1. Give me the things that are X and/or Y.
2. Give me the X things and/or the Y things.
3. Give me the X and/or Y things.

The operations included six disjunction commands, four conjunction commands, and two exclusive-or commands. Half of the commands within each connective-type involved negation.

The commands were as follows:

1. Give me the things that are red and square.
2. Give me the round and black things.
3. Give me the things that are round or green.
4. Give me the black things and the square things.
5. Give me the red or star things.
6. Give me the stars or the green things.
7. Give me the things that are stars and not black.
8. Give me the red and not round things.
9. Give me the things that are red and not square.
10. Give me the round things and the not green things.
11. Give me the square or not green things.
12. Give me the black things or the not star things.

Four different orders of the commands were given.

## RESULTS AND DISCUSSION

### Response Distributions

An analysis of the responses made to the 12 commands is presented in Table 4. The most striking finding for the six positive commands

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Insert Table 4 about here
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is that irrespective of the connective and the idiom used the distribution of responses for different commands is similar; the most frequent response was the intersection of two sets, followed in frequency by one of the

TABLE 4

Distribution of Responses to the Twelve Commands of Experiment 2.

Data Expressed as Percentages.

| Connective | Command | Responses | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $X \cap Y$ | $X \cup Y$ | $X$ | $Y$ | $\frac{1}{n}(X \cap Y)$ | $\frac{1}{n}(X \cup Y)$ | $\frac{1}{n} X$ | $\frac{1}{n} Y$ | All | Misc. |
| $X \cap Y$ | 1 | 42 | 10 | 5 | 7 | 1 | 2 | 12 | 3 | 4 | 14 |
| | 2 | 34 | 10 | 9 | 13 | 0 | 4 | 7 | 2 | 2 | 19 |
| $X \cup Y$ | 3 | 30 | 7 | 16 | 17 | 0 | 2 | 4 | 5 | 6 | 13 |
| | 4 | 21 | 14 | 21 | 3 | 0 | 2 | 13 | 2 | 1 | 23 |
| | 5 | 34 | 6 | 19 | 17 | 0 | 0 | 9 | 1 | 4 | 10 |
| $X$ or $Y$ | 6 | 21 | 6 | 21 | 13 | 0 | 4 | 13 | 7 | 4 | 11 |

| Connective | Command | Responses | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $X \cap \overline{Y}$ | $X \cap Y$ | $X \cap \frac{1}{n} \overline{Y}$ | $X \cup \overline{Y}$ | $X$ | $\overline{Y}$ | $\frac{1}{n} \overline{Y}$ | $Y$ | All | Misc. |
| $X \cap \overline{Y}$ | 7 | 34 | 6 | 24 | 0 | 15 | 4 | 1 | 3 | 1 | 12 |
| | 8 | 26 | 4 | 28 | 0 | 10 | 4 | 2 | 4 | 1 | 21 |
| $X \cup \overline{Y}$ | 9 | 13 | 2 | 28 | 0 | 18 | 4 | 4 | 6 | 3 | 22 |
| | 10 | 17 | 7 | 24 | 0 | 17 | 4 | 2 | 5 | 1 | 23 |
| | 11 | 8 | 12 | 17 | 0 | 15 | 6 | 6 | 4 | 3 | 29 |
| $X$ or $\overline{Y}$ | 12 | 19 | 4 | 24 | 0 | 21 | 6 | 5 | 3 | 1 | 17 |

mentioned sets. However, the connective influenced the responses, for in three of the six commands the most frequent response was the correct one. Furthermore, an examination of the intersection response over all six positive commands revealed that the most frequent intersection response was to an intersection connective. Similarly, the most frequent union response was to a union connective.

The responses to the commands using negation showed the same general trend, with similar distributions of responses even though different idioms and different connectives were used. Unlike the responses to the positive commands where the first- and second-mentioned sets were given with approximately the same frequency, for the negative commands, the first-mentioned set was given significantly more frequently than the second-mentioned set ($\underline{z} = 6.6$, $\underline{p} < .01$). However, since the second-mentioned set was always the complementary set, it is likely that the preference was less a primacy effect than an avoidance of the complementary set.

To test sequence effects, chi-square tests were performed on each of the 12 commands over the four different orders of administering the commands. None of the 12 chi-squares was significant, which indicated that the order of the commands did not affect the number of correct responses.

Regression Models

The four regression models tested were these: a connective model with conjunction, disjunction, negation, and exclusive-or as the variables; an idiom model with one variable corresponding to each of the idioms used; a connective-idiom model with all the idioms and all the connectives of the first two models included; and a connective-interaction model with variables of conjunction, disjunction, exclusive-or, conjunction-negation,

disjunction-negation, and exclusive-or-negation. The results of these four models are summarized in Table 5. The column headed Number of Variables gives the number of variables that entered the stepwise

Insert Table 5 about here

regression with significant effect.

Although the connective-interaction model had the greatest predictive power, it used five variables to predict 12 items. For a very small reduction in predictive power, we may use a two-variable connective model, the variables being disjunction and negation. The results for this connective model are shown in the first line of Table 5. Note that $R^2$ is .867 for this two-variable model, which represents a surprisingly good fit for a model with only three parameters and contrasts sharply with the very bad fit of the idiom model. In the idiom case, only one variable entered the stepwise regression; the remaining variables did not significantly improve the fit.

Although $R^2$ is .867 for the connective model, discrepancies still existed between the predicted and observed probabilities as shown in Table 6.[2]

Insert Table 6 about here

Mathematical Model of Response Distributions[3]

A probabilistic model was constructed that, for a given type of command in Experiment 2, assigns a probability to each type of response. Two types of commands were distinguished: the positive commands (1 to 6) which involve no negation, but only a binary operation, and the negative commands (7 to 12) which involve a negation, as well as a binary operation.

TABLE 5

Summary of Four Regression Models Built

to Predict Correct Response to 12 Commands

| Model | R | $\sigma_e$ | $R^2$ | Number of variables* |
|---|---|---|---|---|
| Connective | .931 | .354 | .867 | 2 |
| Idiom | .067 | .918 | .005 | 1 |
| Connective-Idiom | .936 | .417 | .877 | 5 |
| Connective Interaction | .992 | .154 | .983 | 5 |

*This refers to the number of variables which entered into the model with significant effect.

TABLE 6

Observed and Predicted Probabilities of an Error

Using the Two-Variable Connective Model, Experiment 2.

| Command | Obs. | Pred. | Command | Obs. | Pred. |
|---|---|---|---|---|---|
| 1 | .58 | .47 | 7 | .66 | .83 |
| 2 | .66 | .47 | 8 | .74 | .83 |
| 3 | .93 | .95 | 9 | 1.00 | .99 |
| 4 | .86 | .95 | 10 | 1.00 | .99 |
| 5 | .94 | .95 | 11 | 1.00 | .99 |
| 6 | .66 | .47 | 12 | .73 | .83 |

In order to arrive at a complete response, for a given command, the subject must proceed through several steps. At each step he hands one or two objects to the experimenter. The final answer, or complete response, consists of the set of all objects that the subject gave to the experimenter. Different types of responses correspond to different strategies the subject may adopt.

As a first attempt to apply a quantitative analysis to the types of errors made, we have grouped the response data together independently of the command to which a response was given. Thus, the first assumption of our model is that the strategy adopted is independent of the command. We do not believe this is true when the finer details of the data are examined, but, as we shall see, we can use it in conjunction with some other simplified assumptions to obtain a model that fits the grouped response data fairly well.

The model is based on the subjects' choosing, with probability $p_i$, one of four mutually exclusive and exhaustive response strategies. Strategy 1 is to respond on a random basis. Strategy 2 is to respond as if the binary connective of the command were that of conjunction or intersection. Strategy 3 is to respond to the command by paying attention to only one of the two properties referred to in the command. Strategy 4 is to respond to the command by treating the binary connective as one of disjunction or union.

The properties of our model may be formulated in four simple axioms.

Axiom 1. *The subject chooses with probability* $p_i$, $i = 1, 2, 3,$ *or* $4$, *independent of the actual command, one of the four strategies.*

Axiom 2. *If the subject adopts strategy 3, he considers either property with equal probability.*

Axiom 3. If the command is a negative one, and if the subject adopts strategy 2, 3, or 4, he ignores the negation with probability $\nu$.

Axiom 4. If the subject adopts strategy 2, 3, or 4, with probability $\beta$ he terminates his response sequence before he has given a complete response, except in the case of $X \cap Y$.

The last axiom would be more realistic if probability $\beta$ had been made the parameter of a stopping rule with a truncated geometric distribution, but this more fine-grained assumption cannot be applied to the grouped response data in a direct way, and consequently, the artificially simplified assumption stated in Axiom 4 has been made. Also this stopping rule has not been applied to strategy 2 in the case of positive commands, because $X \cap Y$ consists of only two objects in all cases.

It should be clear how to use the axioms to derive the probability of a response to either a positive or a negative command in terms of the parameters $p_1$, $p_2$, $p_3$, $p_4$, $\beta$, and $\nu$. Using the notation adopted earlier, the probability of responding $\frac{1}{n} X$ to a positive command, for instance, is first the probability $p_3$ of selecting strategy 3 that calls for attending to a single property; then selecting $X$ rather than $Y$, which occurs according to Axiom 3 with probability $\frac{1}{2}$; and finally, terminating the response before it is completed, which occurs with probability $\beta$. Thus, the probability of this response is the product $p_3 \frac{1}{2} \beta$. To take another example, the probability of responding $X \cap \overline{Y}$ to a negative command is first just the probability $p_2$ of selecting strategy 2, then the probability $1 - \nu$ of not ignoring the negation, and finally, the probability $1 - \beta$ of completing the response. Thus, the probability of this response to a negative command is $p_2(1 - \nu)(1 - \beta)$. Using this same method of

analysis, it is easy to obtain the probabilities of each response to the positive or negative commands as shown in the following equations, where $P(\_|P)$ is the conditional probability of a response to a positive command and $P(\_|N)$ to a negative command.

Positive Commands

$$Pr[Misc|P] = p_1$$

$$Pr[X \cap Y|P] = p_2$$

$$Pr[\tfrac{1}{n} X|P] = p_3 \tfrac{1}{2} \beta$$

$$Pr[X|P] = p_3 \tfrac{1}{2} (1-\beta)$$

$$Pr[\tfrac{1}{n} Y|P] = p_3 \tfrac{1}{2} \beta$$

$$Pr[Y|P] = p_3 \tfrac{1}{2} (1-\beta)$$

$$Pr[\tfrac{1}{n} (X \cup Y)|P] = p_4 \beta$$

$$Pr[X \cup Y|P] = p_4 (1-\beta)$$

Negative Commands

$$Pr[Misc|N] = p_1$$

$$Pr[\tfrac{1}{m} X \cap \overline{Y}|N] = p_2 (1-\nu)$$

$$Pr[X \cap \overline{Y}|N] = p_2 (1-\nu)(1-\beta)$$

$$Pr[X \cap \overline{Y}|N] = p_2 \nu$$

$$Pr[\tfrac{1}{m} X|N] = p_3 \tfrac{1}{2} \beta$$

$$Pr[X|N] = p_3 \tfrac{1}{2} (1-\beta)$$

$$Pr[\tfrac{1}{m} \overline{Y}|N] = p_3 \tfrac{1}{2} (1-\nu)\beta$$

$$Pr[\overline{Y}|N] = p_3 \tfrac{1}{2} (1-\nu)(1-\beta)$$

$$Pr[\tfrac{1}{m} Y|N] = p_3 \tfrac{1}{2} \nu\beta$$

$$Pr[Y|N] = p_3 \tfrac{1}{2} \nu (1-\beta)$$

$$Pr[\tfrac{1}{m} X \cup \overline{Y}|N] = p_4 (1-\nu)\beta$$

$$Pr[X \cup \overline{Y}|N] = p_4 (1-\nu)(1-\beta)$$

$$Pr[\tfrac{1}{m} X \cup Y|N] = p_4 \nu\beta$$

$$Pr[X \cup Y|N] = p_4 \nu (1-\beta)$$

Because each of the probabilities shown in the above equations is a simple multinomial, it is easy to derive the maximum-likelihood estimates

of the five parameters. The empirical data from which the estimates were made are shown in Table 7, along with the predicted probabilities. The

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Insert Table 7 about here
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estimates for the five parameters obtained from the data in the table are the following: $\hat{p}_1 = .1867$, $\hat{p}_2 = .3996$, $\hat{p}_3 = .3489$, $\hat{p}_4 = .0648$, $\hat{\beta} = .3754$, and $\hat{\nu} = .1682$. Using these estimates, we computed the predicted probabilities of each type of response to the positive and negative commands. The comparison of the predicted and observed probabilities is shown in Table 7. As can be seen from this table, the fit of the predictions to the data is reasonably good. As is typical in evaluating the goodness of fit of a mathematical model to response data from a large experiment (over 1300 responses), a $\chi^2$ measure-of-fit sharply rejects the null hypothesis, but the qualitative fit is good enough to encourage further model-building efforts in this area.

One weakness of the model is the averaging we have imposed upon the data by ignoring the command itself. Clearly, it is desirable to have a model that takes more account of the fine structure of the data. However, if one examines the current literature in mathematical learning theory, it is apparent that as yet there have not been many theoretical developments that enable models to be applied to data to account for the actual responses made rather than the percentage of errors. In the present case we are especially encouraged, because the types of responses being given by the subjects are complex. In spite of this complexity and the rather large number of different types of responses possible, we have been relatively successful in predicting the patterns of actual responses. The kind of

TABLE 7

Empirical Frequencies, and Observed and Predicted Probabilities of Each Type of Response in Experiment 2.

| Positive Commands | | | | Negative Commands | | | |
|---|---|---|---|---|---|---|---|
| Type | Empirical Freq. | Observed | Predicted | Type | Empirical Freq. | Observed | Predicted |
| Misc. | 129 | .15 | .19 | Misc.' | 122 | .19 | .19 |
| $X \cap Y$ | 204 | .30 | .40 | $\frac{1}{m} X \cap Y$ | 163 | .24 | .12 |
| $\frac{1}{n} X$ | 65 | .08 | .07 | $X \cap \overline{Y}$ | 131 | .20 | .20 |
| $X$ | 101 | .15 | .09 | $X \cap Y$ | 39 | .06 | .07 |
| $\frac{1}{n} Y$ | 22 | .04 | .07 | $\frac{1}{m} X$ | 9 | .01 | .07 |
| $Y$ | 78 | .12 | .11 | $X$ | 107 | .16 | .11 |
| $\frac{1}{n} X \cup Y$ | 14 | .08 | .03 | $\frac{1}{m} \overline{Y}$ | 28 | .03 | .05 |
| $X \cup Y$ | 59 | .09 | .04 | $\overline{Y}$ | 23 | .05 | .09 |
| | | | | $\frac{1}{m} Y$ | 4 | .01 | .02 |
| | | | | $Y$ | 22 | .03 | .02 |
| | | | | $\frac{1}{m} X \cup \overline{Y}$ | 6 | .01 | .02 |
| | | | | $X \cup \overline{Y}$ | 0 | .00 | .03 |
| | | | | $\frac{1}{m} X \cup Y$ | 8 | .01 | .00 |
| | | | | $X \cup Y$ | 0 | .00 | .01 |

experimental situation used in our two studies permits a more detailed recording of data than has been reported here. The experimenter can record the actual sequence of objects given in turn to the experimenter by the subject, and also with some additional effort, the latency of these partial responses. What is most desirable is that subsequent studies attempt to take account of these additional aspects of subjects' responses.

## CONCLUSIONS

The two experiments reported here lead to results that must be regarded as preliminary in character. More extensive and more detailed studies, especially of the language in which logical connectives are embedded, are required before any general conclusions about the comprehension of logical connectives can be drawn. On the other hand, the results of the two experiments, especially the results embodied in the several linear regression models presented in the paper, show that we can account for a large part of the variance in responses of the children by looking at the particular connectives used in a command and also by examining the idiom in which these connectives were expressed. When the idioms are standardized as in the second experiment, the analysis of the connectives alone is sufficient to account for more than 85 per cent of the variance in the behavior. It is to be noted of course that this remark applies to the mean probabilities of response, not to individual responses. Considering the results that have been reported in a wide variety of literature, it is not surprising that negation enters as an important variable in the second experiment. From the studies of Eifermann (1961), Wason (1959, 1961), Wason and Jones (1963), and

others, it might have been predicted that in a regression analysis of comprehension, negation would be a salient connective in terms of difficulty of comprehension. On the other hand, it was unexpected that negation would not be a significant variable in the first experiment. This is probably the result of a considerable dependence between negation and idiom variables, with the idiom variables being somewhat better predictors.

It is easy enough to characterize additional lines of research needed in terms of the comprehension of logical connectives. We have not been able to present in this paper any picture of the developmental sequence in terms of age. It would be desirable to know more about how comprehension changes with age and with linguistic exposure. From a psychological standpoint, it would be especially valuable to have a deeper understanding of why particular sorts of errors were made. The mathematical model we have tested marks only a beginning in this direction.

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## Footnotes

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[2] To avoid problems about the conservation of probability, i.e., to guarantee the predicted $p_i$ always lie between 0 and 1, in Experiment 2 the standard transformation

$$z_i = \log \frac{1-p_i}{p_i}$$

was used, and the regression analysis was made in terms of $z_i$, not $p_i$. The reported $R^2$ is for $z_i$.

[3] The first draft of this section was written in collaboration with Bernard Zarca, but he was not able to participate in writing the final draft of the paper.